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TECHNICAL NOTES

The non-Darcy free convection boundary layer on axi-symmetric and two-dimensional bodies of arbitrary shape

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BOUNDARY layers on bodies immersed in saturated porous media for both free and mixed convection have been the subject of several recent papers and these are reviewed in Cheng [1]. Merkin [2] considered the free convection boundary layers on two-dimensional and axi-symmetric bodies of arbitrary shape embedded in a porous medium under the assumption that the flow was Darcian. Recently Fand *et al.* [3] have investigated experimentally the natural convection heat transfer from a horizontal cylinder which is embedded in a porous medium and they concluded "that the low *Ra* region corresponds to Darcy flow and the high to Forchheimer flow". This was the basis of experimental results for $Ra \lesssim 200$ and therefore we would expect that under the conditions for which the boundary-layer equations are appropriate that the assumption that the flow is Darcian has clearly been violated.

Bejan and Poulikakos [4] reported similarity solutions for vertical boundary-layer natural convection near a solid wall adjacent to a fluid-saturated porous medium in the case when the pore Reynolds number is high enough for the Darcy flow model to break down. Here we will consider the free convection boundary layers on two-dimensional and axi-symmetric bodies of arbitrary shape in a saturated porous medium when the flow is non-Darcian. The bodies are assumed to be impermeable and at a constant temperature which is different to that of the surrounding fluid. We show that the governing equations possess a similarity solution for any body shape and the resulting ordinary differential equation is that as obtained by Bejan and Poulikakos [4].

In the case of a two-dimensional body we consider an infinite cylinder which is placed so that its generators are horizontal and we use the coordinate x to measure the distance round the cylinder from the lowest point. For the axi-symmetric body we place its axis of symmetry vertical and measure x for the lowest point. In both cases the coordinate y is measured normal to the body and $\phi(x)$ is the angle between the outward normal to the body and the downward vertical. We take l to be a typical length scale of the body, T_w and T_∞ are the temperatures of the heated surface and at infinity, respectively, and assume that the boundary-layer thickness is small in comparison to l so that the boundary-layer approximation is applicable. The governing equations can now be written, see Bejan and Poulikakos [4],

$$u+\frac{\rho\chi}{\mu}u^2=\frac{g\beta\kappa\rho}{\mu}(T-T_\infty)\sin\phi \tag{1}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial}{\partial x}(r^n u)+\frac{\partial}{\partial y}(r^n v)=0 \tag{3}$$

where u and v are the fluid speeds in the x and y directions. For two-dimensional bodies $n=0$; and for axi-symmetric bodies $n=1$ and $r(x)$ is the radius of the body; ρ, μ and β are the density, viscosity and the thermal expansion coefficient of the fluid, κ is the intrinsic permeability of the porous medium, α is the thermal diffusivity of the saturated porous medium and g is the gravitational acceleration. The constant χ is the Forchheimer coefficient and equation (1) is the boundary-layer version of the Forchheimer equation. Equations (1)–(3) have now to be solved subject to the boundary conditions

$$\left.\begin{array}{lll} v=0, & T=T_w \text{ on } y=0 & \text{all } x \\ u\to 0, & T\to T_\infty \text{ as } y\to\infty & \text{all } x \end{array}\right\}. \tag{4}$$

If we assume that the flow is Darcian then, following Merkin [2], we find that the x-component of velocity is proportional to the Rayleigh number. Thus within the boundary-layer approximation the non-Darcian term $\rho\chi u^2/\mu$ will dominate the term u in equation (1). Since we know that the non-Darcian term is very important even at Rayleigh numbers of $O(10)$, see Fand *et al.* [3], then we introduce the following non-dimensional quantities

$$\begin{array}{ll} x=Xl, & r=Rl, \\ y=l\left(\dfrac{\chi\alpha}{\nu l}\right)^{1/4}Ra^{-1/4}Y, & u=\left(\dfrac{\alpha\nu}{\chi l}\right)^{1/2}Ra^{1/2}U, \\ v=\dfrac{\alpha}{l}\left(\dfrac{l\nu}{\chi\alpha}\right)^{1/4}Ra^{1/4}V, & T-T_\infty=(T_w-T_\infty)\theta. \end{array} \tag{5}$$

If we introduce the streamfunction ψ, in order to satisfy equation (3), which is defined by

$$U=\frac{1}{R^n}\frac{\partial\psi}{\partial Y},\quad V=-\frac{1}{R^n}\frac{\partial\psi}{\partial X}$$

we can now introduce the following similarity transformation

$$\left.\begin{array}{l} \psi=\left(\displaystyle\int_0^X S^{1/2}(t)R^{2n}(t)\,dt\right)^{1/2}f(\eta),\quad \theta=\theta(\eta) \\ \eta=YS^{1/2}(X)R^n(X)\Big/\left(\displaystyle\int_0^X S^{1/2}(t)R^{2n}(t)\,dt\right)^{1/2}, \\ S(X)=\sin\phi. \end{array}\right\} \tag{6}$$

Equations (1)–(3) and boundary conditions (4) can now be written

$$\theta''+\frac{1}{2}f\theta'=0 \tag{7}$$

$$f'=\theta^{1/2} \tag{8}$$

$$f(0)=0,\quad \theta(0)=1,\quad \theta(\infty)=0 \tag{9}$$

where the primes denote differentiation with respect to η.

Equations (7) and (8) and boundary conditions (9) are identical to those obtained by Bejan and Poulikakos [4] for the boundary-layer flow on a semi-infinite, isothermal, vertical wall in a porous medium for non-Darcian flow. They obtained $\theta'(0)=-0.494$. The local heat transfer

$Q = -k(\partial T/\partial y)_{y=0}$ is then

$$\left.\begin{aligned} Q &= 0.494k(T_w - T_\infty)\frac{S^{1/2}(X)R^n(X)}{\left(\int_0^X S^{1/2}(t)R^{2n}(t)\,\mathrm{d}t\right)^{1/2}} \\ &\quad \times \frac{1}{l}\left(\frac{\nu l}{\chi\alpha}\right)^{1/2} Ra^{1/4} \\ &= 0.494\frac{k(T_w - T_\infty)}{l}\left(\frac{l}{x}\right)^{1/2} Ra^{1/4}\left(\frac{\nu l}{\chi\alpha}\right)^{1/4} \end{aligned}\right\} \quad (10)$$

for a semi-infinite, vertical plate which is identical to the results as obtained in ref. [4]. For a cylinder we set $n = 0$ and $S(X) = \sin X$ whilst for a sphere $n = 1$ and $S(X) = \sin X$.

It is observed from equation (10) that $Q \propto Ra^{1/4}$ whereas for a Darcian fluid $Q \propto Ra^{1/2}$. Fand *et al.* [3] found, experimentally, that for a cylinder the Rayleigh number dependence on the local heat transfer varied from $Ra^{0.694}$ at low Rayleigh numbers to $Ra^{0.372}$ for higher Rayleigh numbers (but still less than about 200). The results presented in this note confirm the conclusions made in ref. [3] that at high Rayleigh numbers non-Darcian effects are very important.

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A further examination of void fraction in annular two-phase flow

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INTRODUCTION AND LITERATURE

A SET of semi-empirical equations for the prediction of void fraction in annular gas–liquid flow had been derived by Tandon *et al.* [1]. These equations are rather cumbersome and consist of three separate equations to cover the entire range of flow rates. This is due to the fact that the derivation had been based on the semi-empirical fragmental representation of velocity distribution due to Von Karman and the Soliman *et al.* [2] curve-fit of the Lockhart–Martinelli data [3].

Butterworth [4] had shown that a number of the more commonly used holdup prediction equations may be represented by the following relationship:

$$\left[\frac{1-\alpha}{\alpha}\right] = A\left[\frac{1-x}{x}\right]^p\left[\frac{\rho_G}{\rho_L}\right]^q\left[\frac{\mu_L}{\mu_G}\right]^r. \quad (1)$$

The homogeneous model, the correlations due to Zivi [5], Turner and Wallis [6], Lockhart and Martinelli [3], Thom [7] and Baroczy [8] may all be shown to be expressible in the form of equation (1).

Chen and Spedding [9] also analysed the idealised annular flow situation and obtained:

$$\alpha = \frac{1}{1+X^{2/3}}. \quad (2)$$

For the case when both the gas and the liquid are flowing in the turbulent regime, X is given by:

$$X_{tt} = (\mu_L/\mu_G)^{0.1}(\rho_G/\rho_L)^{0.5}\left[\frac{1-x}{x}\right]^{0.9}. \quad (3)$$

It was found that equation (2) did not represent well the data available and consequently, an empirical parameter, k, was introduced to result in:

$$\alpha = \frac{k}{k+X^{2/3}}. \quad (4)$$

It is of interest to note at this point that Chen and Spedding [10, 11] analysed the form of equation (1) as given by Butterworth [4] and found that this form of equation may in fact be derived for the case of ideal stratified and ideal annu-

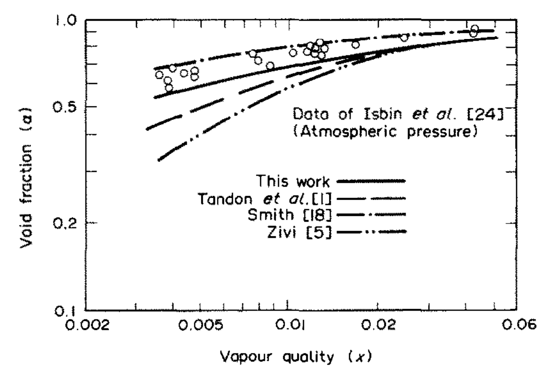

FIG. 1. Comparison of void fraction correlations with experimental data of ref. [24] for the steam–water system at atmospheric conditions.